

**Exam Mesoscopic Physics 18-6-2014 9:00 12:00**

Write each answer on a separate sheet. Indicate for every answer how it is obtained! There are 6 questions

1) (total 15 points)

a) Describe the basic operation of a single electron box. Draw the charge distribution for various relevant values of the gate voltage. (8pts)

A single electron transistor can be used as a sensitive electrometer. It can also be used to read out the charge state of a single electron box.

b) Describe how a single electron transistor can be used to read out the charge state of a single electron box. Show the diagram. Indicate the various capacitances in the system, and estimate (approximately) what their typical values should be. (7pts)

2) (total 15 points)

A one-dimensional (ballistic) wire is made in the form of a closed ring with radius  $R$ . The effective mass is  $m^*$

a) Describe how the energy levels depend on the magnetic flux through the ring. Draw the energy levels as a function of magnetic flux. (3pts)

b) Draw how the persistent current depends on the magnetic flux. Consider 3 situations where the ring contains: only one electron, two electrons,  $N$  electrons (with  $N$  a large number) (3pts)

c) The diameter of the ring is doubled. How does it change the energy levels? (3pts)

d) How does it change the persistent current? (3pts)

e) Explain why an (open ended) ballistic wire cannot support a persistent current, and why a one-dimensional ring can. (3pts)

3) (total 20 points)

Electrons are confined in a semiconductor wire with a circular cross section with diameter  $D$ . The density of electrons (per unit length) is  $n$ . The effective mass is  $m^*$ . Assume that  $D$  is wide enough so that the electronic states can be considered as three-dimensional.

a) Calculate the Fermi energy. (8pts)

When the diameter  $D$  of the wire is reduced, eventually a one-dimensional electron system is obtained.

b) Calculate (or estimate) at which value of  $D$  the system will have become one-dimensional. (7pts)

The wire is now connected to two ideal reservoirs at both ends. Assume that the electron transport through the wire is ballistic. Assume that the diameter of the wire can be changed continuously (e.g. with the electrostatic field of a gate electrode)

c) In this way the number of occupied one-dimensional subbands can be changed. Argue what values will be observed for the quantized steps in the conductance. (5pts)

4) (total 20 points)

a) Give a basic description of the operation of a SQUID (superconducting quantum interference device). Draw the device diagram.(4pts)

b) What is the elementary flux periodicity of a SQUID? (4pts)

The I/V characteristic of a SQUID is measured for different values of the applied magnetic field. The Josephson junctions can be described by the RSJ model.(4pts)

c) Make a schematic drawing of the IV characteristics for two cases: (4pts)

1) The flux through the SQUID is an integer times the superconducting flux quantum

2) The flux through the SQUID is an (integer +  $\frac{1}{2}$ ) times the superconducting flux quantum

Consider the Aharonov-Bohm effect in a normal (metal or semiconductor) ring

d) What are the fundamental flux periodicities? Describe under which conditions they can be observed. (4pts)

e) One of the periodicities is the same as in a SQUID. Is there a difference in the physics? (4pts)

5) (total 20 points)

a) Draw a two-dimensional system which can be used for quantum Hall effect measurements. Also indicate the source and drain contacts, and the contacts used for measurement of the longitudinal and Hall resistances. Indicate the edge channel flow for two occupied Landau levels.(4pts)

b) What is the basic explanation for the quantized Hall resistance? Give an argument why the quantum Hall resistance can be so accurately quantized. (4pts)

c) What can be said about the scattering between edge channels, when the device is under “quantum Hall effect” conditions? (2pts)

d) Because under “quantum Hall effect” conditions the longitudinal resistance becomes zero, the transport is sometimes referred to as “dissipationless”. Is this correct, and is it indeed dissipationless? If not indicate at which locations (and at which contacts) the dissipation takes place. (2pts)

e) Assume that the Fermi energy is located between in between two consecutive Landau levels. A perpendicular magnetic field of 5 Tesla is used. Make an estimate at what temperatures the quantum Hall effect should be measured in order to be accurately quantized. (2pts)

Consider the geometry below. There are 2 Landau levels (and edge) channels occupied. 4 types of scattering processes are illustrated. The first three indicate scattering between edge channels on opposite edges, type 4 indicates scattering between edge channels located on the same edge.

f) The 4 scattering processes are always drawn in pairs (the forward and reverse scattering process) Why? (2pts)

g) Describe and/or argue which one(s) of the four scattering processes affect the accurate quantization of the Hall resistance. (4pts)

6) (total 10 points)

Graphene is the ultimate two dimensional (electronic) system.

a) Describe the similarities in the electronic properties between (single layer) graphene and a two dimensional electron gas formed in a semiconductor hetero structure or quantum well. (5pts)

b) Describe the differences between the two systems. (5pts)